Eur. Phys. J. C 30, 273–277 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01293-2

THE EUROPEAN PHYSICAL JOURNAL C

The Cardy–Verlinde formula and entropy of topological Kerr–Newman black holes in de Sitter spaces

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Received: 7 April 2003 / Revised version: 18 June 2003 /
Published online: 29 August 2003 –

Abstract. In this paper we show that the entropy of a cosmological horizon in 4-dimensional topological Kerr–Newman–de Sitter spaces can be described by the Cardy–Verlinde formula, which is supposed to be an entropy formula of conformal field theory in any number of dimensions. Furthermore, we find that the entropy of a black hole horizon can also be rewritten in terms of the Cardy–Verlinde formula for these black holes in de Sitter spaces, if we use the definition due to Abbott and Deser for conserved charges in asymptotically de Sitter spaces. Such results presume a well-defined dS/CFT correspondence, which has not yet attained the credibility of its AdS analogue.

1 Introduction

Holography is believed to be one of the fundamental principles of the true quantum theory of gravity [1, 2]. An explicitly calculable example of holography is the much-studied AdS/CFT correspondence. Unfortunately, it seems that we live in a universe with a positive cosmological constant which will look like de Sitter space-time in the far future. Therefore, we should try to understand quantum gravity or string theory in de Sitter space preferably in a holographic way. Of course, physics in de Sitter space is interesting even without its connection to the real world; the de Sitter entropy and temperature have always been mysterious aspects of quantum gravity [3].

While string theory successfully has addressed the problem of entropy for black holes, dS entropy remains a mystery. One reason is that the finite entropy seems to suggest that the Hilbert space of quantum gravity for asymptotically de Sitter space is finite dimensional [4, 5]. Another, related, reason is that the horizon and entropy in de Sitter space have an obvious observer dependence. For a black hole in flat space (or even in AdS) we can take the point of view of an outside observer who can assign a unique entropy to the black hole. The problem of what an observer venturing inside the black hole experiences, is much more tricky and has not been given a satisfactory answer within string theory; while the idea of black hole complementarity provides useful clues [6], rigorous calculations are still limited to the perspective of the outside observer. In de Sitter space there is no way to escape the problem of the observer dependent entropy. This contributes to the difficulty of de Sitter space.

More recently, it has been proposed that defined in a manner analogous to the AdS/CFT correspondence, quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space [7] (see also earlier works [8–10]). Following this proposal, some investigations on the dS space have been carried out recently [9–30]. According to the dS/CFT correspondence, it might be expected that as in the case of AdS black holes [31], the thermodynamics of the cosmological horizon in asymptotically dS spaces can be identified with that of a certain Euclidean CFT residing on a spacelike boundary of the asymptotically dS spaces.

One of the remarkable outcomes of the AdS/CFT and dS/CFT correspondence has been the generalization of Cardy's formula (Cardy–Verlinde formula) for arbitrary dimensionality, as well as a variety of AdS and dS backgrounds. In this paper, we will show that the entropy of a cosmological horizon in the 4-dimensional topological Kerr–Newman–de Sitter spaces (TKNdS) can also be rewritten in the form of the Cardy–Verlinde formula. We then show that if one uses the Abbott and Deser (AD) prescription [32], the entropy of black hole horizons in dS spaces can also be expressed by the Cardy–Verlinde formula [33]. In a previous paper [34], we have shown that the entropy of a cosmological horizon in topological Reissner–Nordström–de Sitter spaces in an arbitrary number of dimensions can be described by the Cardy–Verlinde formula. Each of these cases is found to have interesting impli-

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cations in the context of the proposed correspondence. The Cardy–Verlinde formula in the 4-dimensional Kerr–Newman–de Sitter case has been studied previously in [35] (the dS/CFT correspondence having been considered for the 3-dimensional Kerr-de Sitter space already in [36]). In the KNdS case the cosmological horizon geometry is spherical; also the black holes have two event horizons, but in the TKNdS case the cosmological horizon geometry is spherical, flat and hyperbolic for $k = 1, 0, -1$, respectively; also for the case $k = 0, -1$ the black hole does not have an event horizon. On the other hand the entropy of such spaces come from both cosmological and event horizons, and then the absence of an event horizon or the existence of an extra cosmological horizon change the result for the total entropy.

2 Topological Kerr–Newman–de Sitter black holes

The line element of TKNdS black holes in the 4-dimensional case is given by

$$\mathrm{d}s^2 = -\frac{\Delta_r}{\rho^2}\left(\mathrm{d}t - \frac{a}{\Xi}\sin^2\theta\mathrm{d}\phi\right)^2 + \frac{\rho^2}{\Delta_r}\mathrm{d}r^2 + \frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left[a\mathrm{d}t - \frac{(r^2+a^2)}{\Xi}\mathrm{d}\phi\right]^2, \quad (1)$$

where

$$\Delta_r = (r^2+a^2)\left(k - \frac{r^2}{l^2}\right) - 2Mr + q^2,$$
$$\Delta_\theta = 1 + \frac{a^2\cos^2\theta}{l^2},$$
$$\Xi = 1 + \frac{a^2}{l^2},$$
$$\rho^2 = r^2 + a^2\cos^2\theta. \quad (2)$$

Here the parameters M, a, and q are associated with the mass, angular momentum, and electric charge parameters of the space-time, respectively. The topological metric (1) will only solve the Einstein equations if $k = 1$, which is the spherical topology. In fact when $k = 1$, the metric (1) is just the Kerr–Newman–de Sitter solution. Three real roots of the equation $\Delta_r = 0$, are the locations of three horizons, the largest being the cosmological horizon r_{c}, the smallest is the inner horizon of the black hole, the one in between is the outer horizon r_{b} of the black hole.

If we want in the $k = 0, -1$ cases to solve the Einstein equations, then we must set $\sin\theta \to \theta$, and $\sin\theta \to \sinh\theta$ respectively [37–40]. When $k = 0$ or $k = -1$, there is only one positive real root of Δ_r, and this locates the position of cosmological horizon r_{c}. When $q = 0$, $a = 0$ and $M \to -M$ the metric (1) is the TdS (topological de Sitter) solution [43, 44], which has a cosmological horizon and a naked singularity; for this type of solution, the Cardy–Verlinde formula also works well.

Here we review the BBM prescription [23] for computing the conserved quantities of asymptotically de Sitter space-times briefly. In a theory of gravity, mass is a measure of how much a metric deviates near infinity from its natural vacuum behavior; i.e, mass measures the warping of space. Inspired by the analogous reasoning in AdS space [45, 46] one can construct a divergence-free Euclidean quasilocal stress tensor in de Sitter space by the response of the action to variation of the boundary metric; in 4-dimen-sional space-time we have

$$T^{\mu\nu} = \frac{2}{\sqrt{h}}\frac{\delta I}{\delta h_{\mu\nu}} = \frac{1}{8\pi G}\left[K^{\mu\nu} - K\,h^{\mu\nu} + \frac{2}{l}\,h^{\mu\nu} + l\,G^{\mu\nu}\right], \quad (3)$$

where $h^{\mu\nu}$ is the metric induced on surfaces of fixed time, $K_{\mu\nu}$, K are respectively the extrinsic curvature and its trace, and $G^{\mu\nu}$ is the Einstein tensor of the boundary geometry. To compute the mass and other conserved quantities, one can write the metric $h^{\mu\nu}$ in the following form:

$$h_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = N_\rho^2\,\mathrm{d}\rho^2 + \sigma_{ab}\,(\mathrm{d}\phi^a + N_\Sigma^a\,\mathrm{d}\rho)\,(\mathrm{d}\phi^b + N_\Sigma^b\,\mathrm{d}\rho), \quad (4)$$

where the ϕ^a are angular variables parameterizing closed surfaces around the origin. When there is a Killing vector field ξ^μ on the boundary, then the conserved charge associated to ξ^μ can be written as [45, 46]

$$Q = \oint_\Sigma \mathrm{d}^2\phi\,\sqrt{\sigma}\,n^\mu\xi^\nu\,T_{\mu\nu}, \quad (5)$$

where n^μ is the unit normal vector on the boundary, and σ is the determinant of the metric σ_{ab}. Therefore the mass of an asymptotically de Sitter space in four dimensions is

$$M = \oint_\Sigma \mathrm{d}^2\phi\,\sqrt{\sigma}\,N_\rho\,\epsilon \quad ; \quad \epsilon \equiv n^\mu n^\nu\,T_{\mu\nu}\,, \quad (6)$$

where the Killing vector is normalized as $\xi^\mu = N_\rho n^\mu$. Using this prescription [23], the gravitational mass, having subtracted the anomalous Casimir energy, of the 4-dimensional TKNdS solution is

$$E = \frac{-M}{\Xi}. \quad (7)$$

Here the parameter M can be obtained from the equation $\Delta_r = 0$. On this basis, the following relation for the gravitational mass can be obtained:

$$E = \frac{-M}{\Xi} = \frac{(r_{\mathrm{c}}^2+a^2)(r_{\mathrm{c}}^2-kl^2)-q^2l^2}{2\Xi r_{\mathrm{c}}l^2}. \quad (8)$$

The Hawking temperature of the cosmological horizon is given by

$$T_{\mathrm{c}} = \frac{-1}{4\pi}\frac{\Delta_r'(r_{\mathrm{c}})}{(r_{\mathrm{c}}^2+a^2)} = \frac{3r_{\mathrm{c}}^4 + r_{\mathrm{c}}^2(a^2-kl^2) + (ka^2+q^2)l^2}{4\pi r_{\mathrm{c}}l^2(r_{\mathrm{c}}^2+a^2)}. \quad (9)$$

The entropy associated with the cosmological horizon can be calculated as

$$S_{\rm c} = \frac{\pi(r_{\rm c}^2 + a^2)}{\Xi}. \tag{10}$$

The angular velocity of the cosmological horizon is given by

$$\Omega_{\rm c} = \frac{-a\Xi}{(r_{\rm c}^2 + a^2)}. \tag{11}$$

The angular momentum $J_{\rm c}$, the electric charge Q, and the electric potentials $\phi_{q{\rm c}}$ and $\phi_{q{\rm c}0}$ are given by

$$\begin{aligned} \mathcal{J}_{\rm c} &= \frac{Ma}{\Xi^2}, \\ Q &= \frac{q}{\Xi}, \\ \Phi_{q{\rm c}} &= -\frac{qr_{\rm c}}{r_{\rm c}^2 + a^2}, \\ \Phi_{q{\rm c}0} &= -\frac{q}{r_{\rm c}}. \end{aligned} \tag{12}$$

The obtained above quantities of the cosmological horizon satisfy the first law of thermodynamics:

$${\rm d}E = T_{\rm c}{\rm d}S_{\rm c} + \Omega_{\rm c}{\rm d}\mathcal{J}_{\rm c} + (\Phi_{q{\rm c}} + \Phi_{q{\rm c}0}){\rm d}Q. \tag{13}$$

Using (10) and (12) for the cosmological horizon entropy, angular momentum and charge, and also the equation $\Delta_r(r_{\rm c}) = 0$, we can obtain the metric parameters M, a, q as a function of $S_{\rm c}$, $\mathcal{J}_{\rm c}$ and Q, and after that we can write E as a function of these thermodynamical quantities: $E(S_{\rm c}, \mathcal{J}_{\rm c}, Q)$ (see [47, 48]). Then one can define the quantities conjugate to $S_{\rm c}$, $\mathcal{J}_{\rm c}$ and Q, as

$$T_{\rm c} = \left(\frac{\partial E}{\partial S_{\rm c}}\right)_{J_{\rm c},Q}, \quad \Omega_{\rm c} = \left(\frac{\partial E}{\partial J_{\rm c}}\right)_{S_{\rm c},Q},$$
$$\Phi_{q{\rm c}} = \left(\frac{\partial E}{\partial Q}\right)_{S_{\rm c},J_{\rm c}}, \quad \Phi_{q{\rm c}0} = \lim\nolimits_{a\to 0}\left(\frac{\partial E}{\partial Q}\right)_{S_{\rm c},J_{\rm c}}. \tag{14}$$

Making use of the fact that the metric for the boundary CFT can be determined only up to a conformal factor, we rescale the boundary metric for the CFT to the following form:

$${\rm d}s^2_{\rm CFT} = \lim_{r\to\infty} \frac{R^2}{r^2}{\rm d}s^2. \tag{15}$$

Then the thermodynamic relations between the boundary CFT and the bulk TKNdS are given by

$$E_{\rm CFT} = \frac{l}{R}E, \quad T_{\rm CFT} = \frac{l}{R}T, \quad J_{\rm CFT} = \frac{l}{R}J,$$
$$\phi_{\rm CFT} = \frac{l}{R}\phi, \quad \phi_{0\rm CFT} = \frac{l}{R}\phi_0, \tag{16}$$

The Casimir energy $E_{\rm C}$, defined as $E_{\rm C} = (n+1)E - n(T_{\rm c}S_{\rm c} + J_{\rm c}\Omega_{\rm c} + Q/2\phi_{q{\rm c}} + Q/2\phi_{q{\rm c}0})$, and $n = 2$ in this case, is found to be

$$E_{\rm C} = -\frac{k(r_{\rm c}^2 + a^2)l}{R\Xi r_{\rm c}}; \tag{17}$$

in the KNdS space case [35] the Casimir energy $E_{\rm C}$ is always negative, but in the TKNdS space case the Casimir energy can be positive, negative or vanishing, depending on the choice of k. Thus we can see that the entropy (10) of the cosmological horizon can be rewritten as

$$S = \frac{2\pi R}{n}\sqrt{\left|\frac{E_{\rm C}}{k}\right|(2(E - E_q) - E_{\rm C})}, \tag{18}$$

where

$$E_q = \frac{1}{2}\phi_{{\rm c}0}Q. \tag{19}$$

We note that the entropy expression (18) has a similar form as in the case of TRNdS black holes [34].

For the black hole horizon, which only exists for the case $k = 1$ the associated thermodynamic quantities are

$$\begin{aligned} T_{\rm b} &= \frac{1}{4\pi}\frac{\Delta_r'(r_{\rm b})}{(r_{\rm b}^2 + a^2)} \\ &= -\frac{3r_{\rm b}^4 + r_{\rm b}^2(a^2 - l^2) + (a^2 + q^2)l^2}{4\pi r_{\rm b}l^2(r_{\rm b}^2 + a^2)}, \end{aligned} \tag{20}$$

$$S_{\rm b} = \frac{\pi(r_{\rm b}^2 + a^2)}{\Xi}, \tag{21}$$

$$\Omega_{\rm b} = \frac{a\Xi}{(r_{\rm b}^2 + a^2)}, \tag{22}$$

$$\mathcal{J}_{\rm b} = \frac{Ma}{\Xi^2}, \tag{23}$$

$$Q = \frac{q}{\Xi}, \tag{24}$$

$$\Phi_{q{\rm b}} = \frac{qr_{\rm b}}{r_{\rm b}^2 + a^2}, \tag{25}$$

$$\Phi_{q{\rm b}0} = \frac{q}{r_{\rm b}}. \tag{26}$$

Now if we use the BBM mass (7) the black hole horizon entropy cannot be expressed in a form like the Cardy–Verlinde formula [43]. The other way for computing conserved quantities of asymptotically de Sitter space is the Abbott and Deser (AD) prescription [32]. According to this prescription, the gravitational mass of asymptotically de Sitter space coincides with the ADM mass in asymptotically flat space, when the cosmological constant goes to zero. Using the AD prescription for calculating conserved quantities the black hole horizon entropy of TKNdS space can be expressed in terms of the Cardy–Verlinde formula [33]. The AD mass of the TKNdS solution can be expressed in terms of the black hole horizon radius $r_{\rm b}$, a and charge q:

$$E' = \frac{M}{\Xi} = \frac{(r_{\rm b}^2 + a^2)(r_{\rm b}^2 - l^2) - q^2l^2}{2\Xi r_{\rm b}l^2}. \tag{27}$$

The quantities obtained above of the black hole horizon also satisfy the first law of thermodynamics:

$${\rm d}E' = T_{\rm b}{\rm d}S_{\rm b} + \Omega_{\rm b}{\rm d}\mathcal{J}_{\rm b} + (\Phi_{q{\rm b}} + \Phi_{q{\rm b}0}){\rm d}Q. \tag{28}$$

The thermodynamics quantities of the CFT must be rescaled by a factor $\frac{l}{R}$ similar to the previous case. In this case, the Casimir energy, defined by $E'_{\mathrm{C}} = (n+1)E' - n(T_{\mathrm{b}}S_{\mathrm{b}} + J_{\mathrm{b}}\Omega_{\mathrm{b}} + Q/2\phi_{q\mathrm{b}} + Q/2\phi_{q\mathrm{b}0})$, is

$$E'_{\mathrm{C}} = \frac{(r_{\mathrm{b}}^2 + a^2)l}{R\Xi r_{\mathrm{b}}}, \tag{29}$$

and the black hole entropy S_{b} can be rewritten as

$$S_{\mathrm{b}} = \frac{2\pi R}{n}\sqrt{E'_{\mathrm{C}}|(2(E' - E'_q) - E'_{\mathrm{C}})|}, \tag{30}$$

where

$$E'_q = \frac{1}{2}\phi_{q\mathrm{b}0}Q. \tag{31}$$

This is the energy of an electromagnetic field outside the black hole horizon. Thus we demonstrate that the black hole horizon entropy of the TKNdS solution can be expressed in the form of the Cardy–Verlinde formula. However, if one uses the BBM mass (8) the black hole horizon entropy S_{b} cannot be expressed in a form like the Cardy–Verlinde formula. Our result is in favor of the dS/CFT correspondence.

3 Conclusion

The Cardy–Verlinde formula recently proposed by Verlinde [42], relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimensions. In the spirit of the dS/CFT correspondence, this formula has been shown to hold exactly for the cases of the dS Schwarzschild, dS topological, dS Reissner–Nordström, dS Kerr, and dS Kerr–Newman black holes. In this paper we have further checked the Cardy–Verlinde formula with the topological Kerr–Newman de Sitter black hole.

It is well known that there is no black hole solution whose event horizon is not a sphere, in a de Sitter background, although there are such solutions in an anti-de Sitter background; then in TKNdS space for the case $k = 0, -1$ the black hole does not have an event horizon; however the cosmological horizon geometry is spherical, flat and hyperbolic for $k = 1, 0, -1$, respectively. As we have shown there exist two different temperatures and entropies associated with the cosmological horizon and the black hole horizon, in TKNdS space-times. If the temperatures of the black hole and cosmological horizon are equal, then the entropy of the system is the sum of the entropies of cosmological and black hole horizons. The geometric features of the black hole temperature and entropy seem to imply that the black hole thermodynamics is closely related to a non-trivial topological structure of space-time. In [49] Cai et al. in order to relate the entropy with the Euler characteristic χ of the corresponding Euclidean manifolds have presented the following relation:

$$S = \frac{\chi_1 A_{\mathrm{BH}}}{8} + \frac{\chi_2 A_{\mathrm{CH}}}{8}, \tag{32}$$

in which the Euler number of the manifolds is divided into two parts; the first part comes from the black hole horizon and the second part comes from the cosmological horizon (see also [50–52]). If one uses the BBM mass of the asymptotically dS spaces, the black hole horizon entropy cannot be expressed in a form like the Cardy–Verlinde formula [43]. In this paper, we have found that if one uses the AD prescription to calculate conserved charges of asymptotically dS spaces, the TKNdS black hole horizon entropy can also be rewritten in the form of the Cardy–Verlinde formula, which indicates that the thermodynamics of the black hole horizon in dS spaces can also be described by a certain CFT. Our result is also reminiscent of Carlip's claim [53] (to see a new formulation which is free of the inconsistencies encountered in Carlip's work in [54]) that for black holes of any dimensionality the Bekenstein–Hawking entropy can be reproduced using the Cardy formula [55]. Also we have shown that the Casimir energy for a cosmological horizon in the TKNdS space case can be positive, negative or vanishing, depending on the choice of k; by contrast, the Casimir energy for a cosmological horizon in KNdS space is always negative [35].

Acknowledgements. We would like to thank Prof. Robert Mann for useful comments and suggestions.

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